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Maximization of Minimum User Capacity in UAV-Enabled Mobile Networks with NOMA

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***Abstract*—In this letter, we maximize the minimum downlink capacity of moving users in the mobile networks based on non-orthogonal multiple access (NOMA) with flying base stations (FlyBSs) considering practical constraints on the speed, altitude, and transmitting power of the FlyBSs. We propose a geometrical approach allowing us to reflect the users' movement and to derive optimal user clustering for NOMA, positions of the FlyBS, and the transmission power allocation to the users in one cluster served in NOMA at the same time-frequency resources. The proposed solution increases the minimum capacity of all users by 20%-59% comparing to state-of-the-art solutions.**

***Index Terms*—Flying base station, Non-orthogonal multiple access, Minimum capacity, Mobile users, Mobile networks, 6G.**

I. Introduction

In the recent years, unmanned aerial vehicles (UAVs) acting as flying base stations (FlyBSs) have received remarkable interest thanks to their intrinsic characteristics, such as adaptability to environment or potential to improve performance in mobile networks. The advantages offered by the FlyBSs, of course, rely on an efficient radio resource management and FlyBS's positioning. These aspects become even more challenging for non-orthogonal multiple access (NOMA), as the users should be clustered together and the users in each cluster are served at the same radio resources.

Superiority of NOMA over the conventional orthogonal-multiple-access (OMA) is shown in [1]. In [2], an energy-efficient NOMA communication of the FlyBS is further investigated. However, the problem of the FlyBS's positioning is addressed in neither [1] nor [2], as a static FlyBS is assumed. The FlyBS's positioning is targeted, e.g., in [3] and [4] jointly with the transmission power allocation to maximize the sum capacity and to minimize the transmission power, respectively. However, the problem of the NOMA user clustering is not addressed in these papers and only one NOMA cluster is considered. The assumption of single cluster is practical for only scenario with few users due to a high complexity of successive interference cancellation (SIC) decoding commonly adopted to cancel interference within the NOMA cluster [5].

Furthermore, in [6], the authors minimize an energy consumption of the IoT devices in scenarios with the FlyBS collecting data from these devices in NOMA networks. To this end, the FlyBS's trajectory and resource allocation are optimized. Nevertheless, the trajectory is designed for the users being static during the FlyBS's entire mission and an extension towards mobile users is not straightforward.

The mobile users are considered in [7], where joint NOMA user clustering and FlyBS's positioning is investigated to prolong the coverage duration of the users served by the FlyBS. The solution in [7], however, targets 1D scenario, where all users move in the same direction along x-axis (e.g., vehicles on a road). Hence, the clustering in [7] is done based on only the x-coordinate of the users. Then, a heuristic solution for a joint user clustering and FlyBS's positioning to increase the minimum downlink sum capacity is proposed in [8]. The NOMA clusters size is, however, limited to only two users. Furthermore, the solution in [8] does not guarantee any minimum capacity to individual users, hence, some users may end up with zero capacity. The problem of NOMA clustering in the scenario with potentially moving users targeting to minimize the transmission power and to maximize the sum capacity is addressed in [10] and [11], respectively. However, the FlyBS's positioning is considered in neither [10] nor [11], as only a static base station is assumed in both.

Even if the problem of maximization of the minimum capacity for users reflects fairness among users and is heavily addressed in common mobile networks, it is not yet investigated for the NOMA-based networks with FlyBSs serving mobile users. Hence, we maximize the minimum capacity via a joint FlyBS's positioning, transmission power allocation, and NOMA users clustering in the scenario with the moving users. In addition, we consider a generalized model for NOMA, where the cluster sizes can be different and the cluster size of one is allowed, i.e., some users can potentially be served in OMA. We derive a closed-form expression for the transmission power in terms of the FlyBS's position and the NOMA clustering. We also derive necessary conditions for the users' capacities so that the minimum capacity of all users can be maximized. Then, we maximize the minimum capacity via the FlyBS's positioning and the transmission power allocation to the users and we find the optimal user clustering.

II. System model and problem formulation

In this section, we explain the system model and then formulate the problem of user clustering, power allocation, and FlyBS's positioning.

We consider one FlyBS serving N_u mobile users $U = \{u_1, u_2, \dots, u_{N_u}\}$ as depicted in Fig. 1. In our NOMA model, the users are assigned into different clusters such that the users in each cluster share the same channel at the same time,

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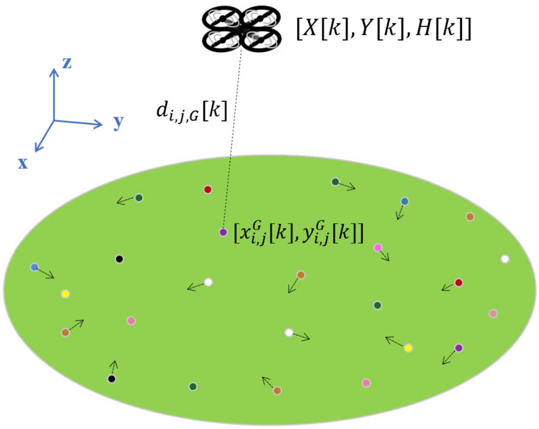

Fig. 1: System model with multiple mobile users deployed within coverage area of the FlyBS. The circles with the same color represent the users in the same NOMA cluster.

however different clusters are served at different (orthogonal) channels. Let $\mathbb{G}$ denote the space of all possible functions that group the users into N_{cl} clusters with $N_{cu,j}$ users in the j-th cluster. We assume that $N_{cu,j} \in [1, N_{cu}^{max}]$, where N_{cu}^{max} is the maximum potential size of each cluster and it is practically related to an incurred complexity in the SIC decoder. Each function $G \in \mathbb{G}$ is defined as a bijective mapping G: $[1, N_u] \rightarrow [1, N_{cl}] \times [1, N_{cu}^{max}]$. Furthermore, let $u^G_{1,j}, u^G_{2,j}, \dots, u^G_{N_{cu,j},j}$ denote the users assigned by the function G to cluster j. Let $\boldsymbol{l_M}[k] = \left[X[k], Y[k], H[k]\right]^T$ and $\boldsymbol{l^G_{i,j}}[k] = \left[x^G_{i,j}[k], y^G_{i,j}[k]\right]^T$ denote the locations of the FlyBS and $u^G_{i,j}$, respectively, at the time step k . In SIC, suppose that the user $u^G_{i,j}$ ($i \in [1, N_{cu,j} - 1]$) cancels the interfering signals from the users $u^G_{i',j}$ ($\forall i' \in [i+1, N_{cu,j}]$) to extract its own signal (note that the adopted decoding order would not affect optimality since finding the index of users in clusters is subject to optimization instead). Consequently, the achievable SINR $\gamma^G_{i,j}$ for $u^G_{i,j}$ is:

$$\gamma^G_{i,j}[k] = \frac{p^{G,R}_{i,j}[k]}{\sigma^2 + \sum_{l=1}^{i-1} p^{G,R}_{i,l,j}[k]}, \quad (1 \leq i \leq N_{cu,j}), \quad (1)$$

where σ^2 is the noise power, $p^{G,R}_{i,j}$ is the received power by $u^G_{i,j}$, and $p^{G,R}_{i,l,j}$ represents the interference at $u^G_{i,j}$ caused by the signals transmitted to another user $u^G_{l,j}$ in the same cluster j.

The channel capacity $C^G_{i,j}$ of the user $u^G_{i,j}$ is calculated as $C^G_{i,j}[k] = B\log_2(1 + \gamma^G_{i,j}[k])$, where B is the bandwidth assigned to each NOMA cluster.

Next, we formulate the FlyBS's total transmission power as $P_{TX}(\boldsymbol{l}, k, G) = \sum_{j=1}^{N_{cl}} \sum_{i=1}^{N_{cu}} p^{G,T}_{i,j}$, where $p^{G,T}_{i,j}$ is the transmission power of the FlyBS to the i-th user $u^G_{i,j}$ in the j-cluster calculated by the Friis' transmission equation as:

$$p^{G,T}_{i,j} = \frac{\zeta}{(\frac{\lambda}{\lambda+1}\bar{\epsilon} + \frac{1}{\lambda+1}\tilde{\epsilon})} p^{G,R}_{i,j} d^{\alpha}_{i,j,G}[k] = Q p^{G,R}_{i,j} d^{\alpha}_{i,j,G}[k], \quad (2)$$

where $p^{G,R}_{i,j}$ is the received signal power by the user $u^G_{i,j}$, $d_{i,j,G}$ denotes the distance between the FlyBS and the user $u^G_{i,j}$, ζ is a parameter depending on communication frequency and gain of antennas. Furthermore, λ is the Rician fading factor, $\bar{\epsilon}$ is the line-of-sight (LoS) component satisfying $|\bar{\epsilon}| = 1$, and $\tilde{\epsilon}$ denotes the non-line-of-sight (NLoS) component satisfying $\tilde{\epsilon} \sim CN(0, 1)$, and α is the pathloss exponent.

Using (2), the total transmission power P_{TX} is rewritten as: $P_{TX}(\boldsymbol{l_M}, k, G) = \sum_{j=1}^{N_{cl}} \sum_{i=1}^{N_{cu,j}} Q p^{G,R}_{i,j} {d_{i,j,G}}^{\alpha}[k]$.

Our goal is to find the position of the FlyBS jointly with the transmission power allocation to the users and the NOMA clustering of the users to maximize the minimum capacity $\eta[k]$ at every time step k. We formulate this problem as:

$$\max_{\left[G, p^{G,T}_{i,j}, \boldsymbol{l_M}\right]} \eta[k], \quad (3)$$

$$s.t. \quad C^G_{i,j}[k] \geq \eta[k], \quad (3a)$$

$$P_{TX}(\boldsymbol{l_M}, k, G) \leq P^{max}_{TX}, \forall k, \quad (3b)$$

$$H_{min} \leq H[k] \leq H_{max}, \quad (3c)$$

$$\left|\left|\boldsymbol{l_M}[k] - \boldsymbol{l_M}[k-1]\right|\right| \leq V_{max}\delta_k, \quad (3d)$$

where δ_k in (3d) is the duration between time steps $k-1$ and k. The constraint (3b) guarantees that the FlyBS's transmission power is within the maximum transmission power limit P^{max}_{TX}, and the constraints (3c) and (3d) limit the FlyBS's altitude to $[H_{min}, H_{max}]$ and the FlyBS's speed to V_{max}.

III. Proposed FlyBS Positioning, Transmission Power Allocation, and User Clustering

In this section, we first transform of the problem to more convenient transmission power minimization. Then, we describe the proposed users clustering and FlyBS positioning.

A. Maximization of minimum capacity via minimization of transmission power

The user's capacity and so the minimum capacity (objective) in (3) are not convex or concave. Furthermore, the discrete function G makes the problem (3) non-tractable. To tackle these challenges, we propose a solution based on a conversion of the objective in (3). To this end, we first elaborate the characteristics of the optimal solution to (3) (denoted by η^*) and its relevance to the transmission power via Proposition 1.

Proposition 1. *In the optimal solution to (3), $C^G_{i,j} = \eta^*, \forall i, j$.*

Proof. We first show that P_{TX} is an increasing function of $C^G_{i,j}$. To this end, from (1) and (2), we observe that $p^{G,T}_{1,j} = Q\sigma^2\gamma^G_{1,j}[k]d^{\alpha}_{1,j,G}[k]$. Thus, $p^{G,T}_{1,j}$ increases with $\gamma^G_{1,j}[k]$. Furthermore, from (1) and (2), $p^{G,T}_{i,j}$ is rewritten as:

$$p^{G,T}_{i,j} = \sigma^2\gamma^G_{i,j}[k]d^{\alpha}_{i,j,G}[k] + \gamma^G_{i,j}[k] \sum_{l=1}^{N_{cu,j}-1} p^{G,T}_{l,j}. \quad (4)$$

From the recursive formula in (4), we observe that $p^{G,T}_{i,j}$ also increases with $\gamma^G_{i,j}$ and, consequently, with $C^G_{i,j}$ (since $\gamma^G_{i,j}[k] = 2^{\frac{C^G_{i,j}[k]}{B}-1}$). Now, by contradiction, suppose that the capacity received by some user $u^G_{i',j'}$ would be greater than η^* in the optimal solution to (3). In such case, decreasing $C^G_{i',j'}$

as long as it still remains larger than or equal to η^* would not decrease the minimum capacity η^* while it decreases the transmission power. This means that, given the maximum transmission power limit P_{TX}^{max}, we can always first reduce the capacity of the users to η^* (to reduce the transmission power) and, then, we can increase the capacity of all users together by increasing the power to all users (exploiting the reduced transmission power) until the transmission power reaches the P_{TX}^{max}. This proves the Proposition 1. □

It is deduced from the proof of the Proposition 1 that, once the optimal G and $\boldsymbol{l_M}$ are derived, the optimal power allocation $p_{i,j}^{G,T}$ in (3) is determined from (4) and by setting the FlyBS's total transmission power to P_{TX}^{max}. Furthermore, Proposition 1 indicates the connection between the maximization of η^* and the minimization of P_{TX} as elaborated and demonstrated in the following Corollaries.

Corollary 1. *Using the expression for $p_{i,j}^{G,T}$ in (4) and by the fact that the capacity of all users is η^* and $\gamma^*[k] = 2^{\frac{\eta^*[k]}{B}} - 1$, the total transmission power is calculated as:*

$$P_{TX}(\boldsymbol{l_M}, k, G) = \gamma^* Q \sigma^2 \sum_{j=1}^{N_{cl}} \sum_{i=1}^{N_{cu,j}} (1+\gamma^*)^{N_{cu,j}-i} d_{i,j,G}^{\alpha}, \quad (5)$$

Corollary 2. *Following the Proposition 1 and given that the transmission power increases with the capacity η^* and vice versa, the solution to the user clustering and FlyBS's positioning in (3) can be alternatively derived via solving the following problem of transmission power minimization:*

$$\min_{[G, \boldsymbol{l_M}]} \; P_{TX}[k], \forall k, \quad (6)$$

$$s.t. \quad (3c), (3d),$$

$$C_{i,j}^{G}[k] = \eta_{arb}, i \in [1, N_{cu,j}], \; j \in [1, N_{cl}], \quad (6a)$$

where η_{arb} is an arbitrary user capacity. The constraint (6a) *ensures that every user receives the capacity of exactly η_{arb}.*

Note that the value of η_{arb} does not affect the optimal G or $\boldsymbol{l_M}$ in (6) (see Proposition 1), but it is selected such that the constraint (3b) would not be violated (so that there would exist a feasible solution). The power minimization problem in (6) is more convenient for FlyBS's positioning and user clustering, because the maximization of minimum capacity requires the knowledge of which user receiving the smallest (minimum) capacity among all users. However, such knowledge is hard to obtain without determining the user clustering. On the other hand, the objective in (3) and the constraints (3c) and (3d) are convex with respect to $\boldsymbol{l_M}$. Hence, for a fixed clustering function G, the optimal FlyBS's position is efficiently derived using CVX. Note that (6a) is translated into $\gamma[k] = 2^{\frac{\eta_{arb}[k]}{B}} - 1$, which is used to write the transmission power as in (5).

B. Determination of user clustering and FlyBS positioning

Although the optimal position for each clustering can be derived (as explained in the previous subsection), finding the optimal clustering by an exhaustive search is not always feasible, because, even if all the clusters would be of the same size, there are $\frac{N_u!}{N_{cl}!}$ different clustering options, which could be extremely large even for small N_u.Hence, to tackle the user clustering, we derive all *promising* clustering options to reduce complexity and we select the one maximizing the performance. First, let's define a necessary condition for the optimal clustering. To do this, we introduce the following terminology. Let Λ denote a set of points in the xy-plane. The subset $T \subset \Lambda$ is said to form an "exclusively convex polygon" if the convex hull determined by T does not enclose any point from $\Lambda - T$. Furthermore, let Ψ denote the set of points in the xy-plane corresponding to the users' locations. Now, we define the necessary condition via Theorem 1.

Theorem 1. *Denoted by $\Theta_{e,q}$ the set of e-th users in clusters of size q, for the optimal clustering G_{opt} minimizing P_{TX}, $\Theta_{1,N_{cu}^{max}}$ forms an exclusively convex polygon in Ψ. Moreover, $\Theta_{2,N_{cu}^{max}} \cup \Theta_{1,N_{cu}^{max}-1}$ is an exclusively convex polygon in $\Psi - \Theta_{1,N_{cu}^{max}}$. Similarly, $\Theta_{1,N_{cu}^{max}-i} \cup ... \cup \Theta_{i,N_{cu}^{max}-1}$ is an exclusively convex polygon in the set of unselected users, i.e., $\Psi - \cup_{r=1}^{i} \cup_{j=0}^{r-1} \Theta_{r,N_{cu}^{max}-j}$ for $(1 \leq i \leq N_{cu}^{max} - 1)$.*

Proof. Let $N_{cl,i}$ denote the number of clusters of size i and $cl_{j_1}, ..., cl_{j_{N_{cl,N_{cu}^{max}}}}$ be the clusters of size N_{cu}^{max}. Suppose that $\Theta_{1,N_{cu}^{max}} = \{u_{1,cl_{j_1}}^{G_{opt}}, u_{1,cl_{j_2}}^{G_{opt}}, ..., u_{1,cl_{j_{N_{cl,N_{cu}^{max}}}}}^{G_{opt}}\} = \{u_{i_1}, ..., u_{i_{N_{cl,N_{cu}^{max}}}}\}$. By contradiction, we assume there exists user u_j such that u_j belongs to the convex hull for $\{u_{i_1}, ..., u_{i_{N_{cl,N_{cu}^{max}}}}\}$, but $u_j \notin \{u_{i_1}, ..., u_{i_{N_{cl,N_{cu}^{max}}}}\}$. Furthermore, suppose that the convex polygon's vertices are $\{u_{v_1}, ..., u_{v_q}\}$. For G_{opt}, we assume that u_j is the user at the s-th position in some cluster. Let G_l denote the clustering function obtained by swapping u_{v_l} from the polygon and u_j in G_{opt} for $l \in [1, q]$. From the optimality of G_{opt}, it is inferred that $(P_{TX}(G_{opt}) - P_{TX}(G_l)) < 0$. Now, we rewrite $(P_{TX}(G_{opt}) - P_{TX}(G_l))$ at (X_{opt}, Y_{opt}) by means of the system parameters as:

$$\gamma^{arb} Q \sigma^2 (d_{i_l}^{\alpha} - d_j^{\alpha})((1+\gamma^{arb})^{N_{cu}^{max}-1} - (1+\gamma^{arb})^{N_{cu}^{max}-s}) \quad (7)$$

From the inequality $P_{TX}(G_{opt}) - P_{TX}(G_l) < 0$ and from (7), it is concluded that $d_{i_l} - d_j < 0$ for $l \in [1, q]$. Since the points in $\{u_{v_1}, ..., u_{v_q}\}$ create the convex polygon, the set of inequalities $d_{i_l} - d_j > 0$ also demarcates the convex polygon Γ_i. Now, for any $w \in [1, q]$, let's consider the edge from Γ_i corresponding to $d_{i_w} = d_j$. Since Γ_i is convex, all the vertices in Γ_i lie in the halfplane defined by $d_{i_w} > d_j$. This implies that the intersection of the inequalities $d_{i_l} - d_j < 0$ $(\forall l \in [1, q])$ is an empty set. This contradicts the initial assumption that the points corresponding to the users in $\Theta_{1,N_{cu}^{max}}$ do not create the exclusively convex polygon. By a similar procedure, the second users of the clusters of size N_{cu}^{max} and the first users of the clusters of size $N_{cu}^{max} - 1$ should also create an exclusively convex polygon in the set remained from Ψ, and so on so forth for the next users of all clusters. This completes the proof. □

In order to apply Theorem 1, we now find all subsets of the users forming an exclusively convex polygon. We develop our algorithm based on a method provided in [9], where general

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convex polygons (CPs) are found based on a target vertex size. The solution in [9] cannot be immediately used in our case, as we need to find CPs with a certain total number of points belonging either to the vertices or to the interior of the CP. Hence, we extend the general solution in [9] to enable a selection of the exclusively convex polygons of an arbitrary size m from the set Ψ, see Algorithm 1. The set of points Ψ represents the locations of the users in the xy-plane. Following [9], the principle of finding such CPs is to first set the lowermost point in the polygon (line 2 in Algorithm 1) and, then, create sequences of vertices by collecting all next candidate points (users) to be included in the sequence (lines 3 and 5). The addition of the points to the sequence should not violate the convexity presumption of the polygon (line 5). After adding new vertex to our sequence of vertices (line 9), we check the number of enclosed points by the CP of the sequence (lines 7 and 12). Then, sequences containing more points than m are excluded from the vertex search.

Following the proposed search for polygons in Algorithm 1, we now find the optimal clustering using Theorem 1. Note that, in Theorem 1, the size of the clusters is prespecified. Hence, we should first find all possible sets of the cluster sizes $\{N_{cu,1},\dots,N_{cu,cl}\}$ with a maximum cluster size of N_{cu}^{max}. Then, for each derived set $\{N_{cu,1},\dots,N_{cu,cl}\}$, we apply Theorem 1 as follows. To determine the first users in the NOMA clusters with a size of N_{cu}^{max}, we execute Algorithm 1 for $m = N_{cl,N_{cu}^{max}}$ and collect all candidate sets of users. Next, for each obtained candidate set, we determine the second users in the clusters with a size of N_{cu}^{max} as well as the first users in the clusters with a size of $N_{cu}^{max}-1$. To this end, we apply Algorithm 1 for $m = N_{cl,N_{cu}^{max}} + N_{cl,N_{cu}^{max}-1}$ over the set of the remaining users and so on and so forth until every user is assigned to one cluster.

Algorithm 2 summarizes the whole proposed solution to (3). First, the optimum position of the FlyBS is determined (line 1) using CVX for each clustering derived by Algorithm 1. Next, the clustering and corresponding position yielding the smallest transmission power are selected (lines 2-3) and the transmission power is allocated (line 4) via (4).

The computational complexity of the proposed solution is determined by the number of clustering candidate options and by the calculation of the transmission power and the FlyBS's position from the simulations. By caluclating the order of complexity with respect to the system parameters, the total computational complexity is $O({N_{cl}}^{0.83N_{cu}^{max}}{N_u}^{1.81N_{cu}^{max}})$. Despite the exponential complexity with respect to N_{cu}^{max}, complexity is still low and allows a fast enough processing for practical applications, as N_{cu}^{max} is relatively low in real-world applications due to an implementation complexity of SIC for large N_{cu}^{max}.

IV. Simulations and results

In this section, we provide details of models and setting adopted for the performance evaluation and we demonstrate the advantages of the proposal over state-of-the-art schemes.

Algorithm 1 Find all exclusively convex polygons of size m

$U_q \subset \Psi$: halfspace of points with y-coordinate larger than that of q
$H_{q,w}$: halfspace to the left of the directed line qw
$S = \Psi$: initial set for the lowermost point when counting polygons
$\Omega_s \leftarrow []$: Matrix to store sequences of vertices on top of s
$CP(\Phi)$: convex polygon (CP) obtained from the vertices Φ

1: **while** $S \neq \emptyset$ **do**
2: $s \leftarrow$ point with smallest y-coordinate in S, $\Omega_s \leftarrow [\Omega_s, s]$
3: **for** every point $q \in U_s$ **do** add q to every row in Ω_s
4: delete rows in Ω_s whose size of CP is larger than m
5: **for** every $w \in U_s \cap H_{s,q}$ **do** add w to every row in Ω_s
6: **for** $j = 1$, $j \leq$ rows(Ω_s), $j++$ **do**
7: delete row j if $|CP(\Omega_s(j,:))| > m$
8: **end for**
9: $q \leftarrow w$
10: **end for**
11: **end for**
12: delete rows in Ω_s whose size of CP is not equal to m
13: $S \leftarrow S - \{s\}$
14: **end while**

Output: Ω_s, $s \in \Psi$: vertex sequences of convex polygons of size m

A. Simulation scenario and models

We consider a scenario with 60–240 active users in an outdoor event (sports, festivals, etc.) within 500 m×500 m area. The users are assumed to leave the area through four exit paths in the ± directions of the x and y axes with $N_u/4$ users on each path. We consider three crowds on each path with $N_u/12$ users in each crowd with the speed of users in the first, second, and third crowds uniformly distributed over intervals of [0.6, 1.4] m/s, [1, 2] m/s, and [1.5, 2.5] m/s, respectively.

Following [8] we assume $\alpha = 2$. Omni-directional antennas with gains of 7 dBi and 0 dBi for the FlyBS and the users are considered, respectively. The radio frequency of 2.6 GHz and the bandwidth of 100 MHz are selected. Note that the bandwidth is equal for all clusters. Spectral density of noise is set to -174 dBm/Hz. The allowed range for the FlyBS's altitude is H_{min} =150 m and H_{min} =350 m. The maximum transmission power P_{TX}^{max} is set to 1 W. Each simulation lasts for 1200 seconds with the problem (3) solved every second. The results are averaged out over 100 simulation drops.

We compare our proposal with following state-of-the-art works: i) maximization of the minimum sum capacity among all clusters via FlyBS's positioning and NOMA pairing [8] (labeled as *max-min-C*), ii) transmission power minimization via NOMA clustering for static base stations (SBSs) [10] (*min-Tx*), iii) sum capacity maximization via NOMA clustering for the SBS [11] (*max-C*), iv) enhanced version of [10] with our proposed FlyBS positioning (*E-min-Tx*), and v) enhanced

Algorithm 2 Optimal user clustering and FlyBS positioning

1: derive optimal $\boldsymbol{l_M}[k]$ via CVX for each clustering from Alg. 1
2: calculate P_{TX} for each clustering according to (5)
3: optimal $G \leftarrow \text{argmin}_G\, P_{TX}$
4: calculate $p_{i,j}^{G,T}$ from (4)

Output: optimal $\left[G, p_{i,j}^{G,T}, \boldsymbol{l_M}\right]$

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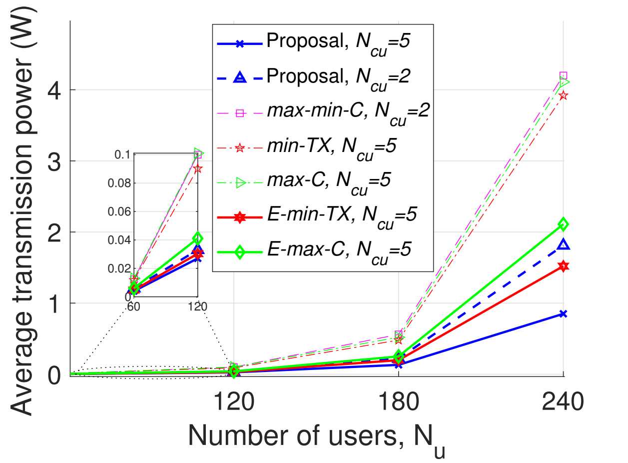

Fig. 2: Transmission power required for N_u users, $C^*_{arb} =$ 4 Mbps.

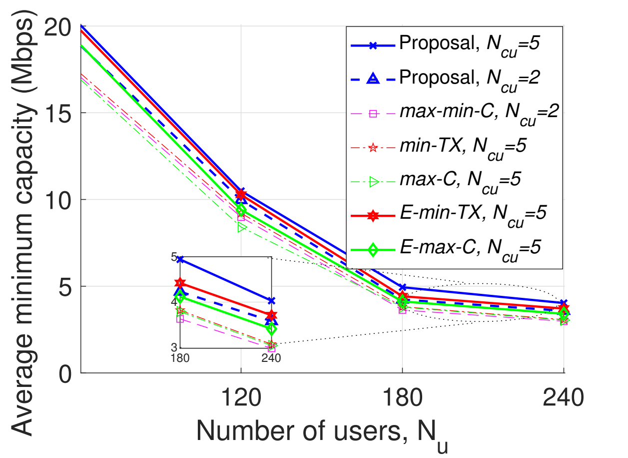

Fig. 3: Average minimum capacity achieved by all N_u users.

version of [11] with our proposed positioning (*E-max-C*).

B. Simulation results

In this subsection, we present and discuss simulation results. Fig. 2 demonstrates the transmission power increases with the number of users, because less bandwidth is available for each user and a larger transmission power is required (see (4)). All benchmarks require notably larger transmission power compared to our proposed solution. As *max-min-C* supports only $N_{cu}^{max} = 2$, we compare it with the proposed solution for $N_{cu}^{max} = 2$ and the proposal reduces the transmission power by 67%. By an extension of the cluster size to $N_{cu}^{max} = 5$, the transmission power required by our proposal is further reduced by 54% compared to the proposal with $N_{cu}^{max} = 2$. This is because for a larger cluster size, fewer clusters are created and more bandwidth can be allocated to each cluster resulting in a lower transmission power. Overall, the proposed solution (with $N_{cu}^{max} = 5$) reduces the transmission power by 78% and 80% with respect to *min-Tx*, *max-C*, respectively. The extension of *min-Tx* and *max-C* with our proposed FlyBS positioning towards *E-min-TX* and *E-max-C* demonstrates that our positioning reduces the transmission power of the original *min-TX* and *max-C* by 67% and 59%, respectively.

Fig. 3 shows that the average minimum capacity η^* decreases with the number of users, because the more users are served by the FlyBS, the narrower channel is allocated to each user, as the whole available bandwidth is split among clusters. The proposed solution improves the average η^* compared to *max-min-C* by up to 20% (for $N_{cu}^{max} = 2$). In addition, the average η^* is enhanced by 17% for our proposal if N_{cu}^{max} is increased to 5. In total, compared to *max-min-C*, *min-Tx*, *max-C*, *E-min-Tx*, *E-max-C* the proposed solution (for $N_{cu}^{max} = 5$) increases the average η^* by 35%, 44%, 59%, 11%, and 20%, respectively.

V. Conclusions

In this letter, we have provided a geometrical approach to derive NOMA clustering, transmission power allocation to the users, and the FlyBS's positioning maximizing the minimum capacity among all users. We have shown that the proposed solution enhances the minimum capacity by tens of percent with respect to state-of-the-art work. In the future, the scenario with multiple FlyBSs shall be studied.

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